

## In-plane buckling analysis of transversely loaded timber beams

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The loadbearing capacity of a transversely loaded timber beam is today in structural design determined by use of a strength criterion  $\sigma_m \leq f_m$ . The bending strength  $f_m$  is considered as a material parameter representative for the wooden material without considering any influence of second order effects.

It is well known that buckling should be considered in case of compressive axial loading but that this also holds for transversely loaded beams is not yet recognised as for the cantilever beam shown below.

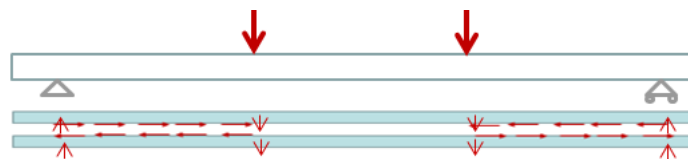


A new model is presented in this paper, dealing with timber beams subjected to transversal loads. For simplicity only in-plane displacements will be considered. Two different kinds of buckling modes (orthogonal eigenmodes) of great importance can appear and both should be checked in order to determine the loadbearing capacity.

Greenhill was first to solve the problem of finding the buckling load for a column subjected to a uniformly distributed dead load. He used Bessel functions, see [1] and [2]. Nowadays this type of problem can be solved by finite element analysis considering it as an eigenvalue problem. The most important material parameter will be the bending stiffness as in case of ordinary Euler buckling. The second kind of buckling modes refers to cases where the loading mainly causes bending deformations. Buckling will occur for a much lower load value than expected with respect to the measured value of the bending stiffness. The existence of this second kind of buckling seems to be unknown in literature.

To explain the buckling phenomenon of the second kind the beam is thought to be lengthwise subdivided into two halves, one mainly in compression and the other one in tension. The fictitious cut dividing the beam into two parts is along the neutral axis. For statically determined beams we can directly find the shear stresses acting on the cutting planes of the two beam parts. The shear stresses can be considered as external axial loads giving rise to an antisymmetric buckling mode. The critical load value when buckling occurs is obtained by an eigenvalue analysis. The lowest eigenvalue will be the same for the compressed and tensioned parts except for the sign.

The example below shows the main features in the modelling for solving beam buckling problems of the second kind. The specimen studied is given dimensions according to EN 408 and is of special interest as it is used in classifying timber into different strength classes. The validity of the proposed theory is supported by results from test series with more than 500 boards of different dimensions [3].



When the E-modulus is constant over the cross section the total loadbearing capacity is three times that of one individual beam part. This assumption of a constant stiffness over the cross section will often result in misleading results. A much better strength prediction is normally obtained if the radial variation of the longitudinal E-modulus from pith to bark is considered.

### References

- [1] S. P. Timoshenko and J. M. Gere: Theory of Elastic Stability.
- [2] A. G. Greenhill: Proc. Cambridge Phil. Soc., Vol. 4, 1881.
- [3] H. Petersson, B. Källsner and S. Ormarsson: Strength grading of structural timber based on buckling analysis and scanning techniques, In WCTE, Vienna Austria, 2016.