

# Approximation of stresses in multi-span CLT beams based on refined zigzag theory

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In the recent years Cross Laminated Timber (CLT) has become an indispensable engineered wood product, especially within Europe. Due to its shear-flexible transverse layers, the calculation method has to capture these effects. Most commonly used approximation methods in practice are the Timoshenko beam theory, the gamma-method and the shear analogy method. Each method has its advantages and shortcomings. A comprehensive study on uniformly loaded multi-span beams using the mentioned methods was conducted in [1]. It was shown, that at the middle supports (local force introduction) the normal stresses (Timoshenko and gamma method) deviate from the reference solution (2D FE solution). The shear analogy method was able to capture these local effects better than the other employed methods, but with a higher modelling effort (two coupled beam elements). Higher beam theories are a possibility to reduce the modelling and discretization effort and reach a suitable approximation of normal stresses within CLT at middle support areas.

This contribution deals with the application of the Refined Zigzag Theory (RZT) for calculating stresses in CLT elements under local load introduction (eg middle supports in multi-span beams). RZT is a robust displacement theory, well suited for beam and plate finite elements, that has been developed recently [2]. Refined zigzag theory makes use of Timoshenko as its baseline and a product of the zigzag function  $\psi^{(k)}(z)$  and the amplitude of the zigzag displacement  $\phi(x)$  is added. The zigzag function is predefined through the thickness and the shear modulus of the lamellas. Adding one additional degree of freedom in case of an 1D beam.

Within this contribution an analytic solution of RZT [3], as well as RZT-FE-beam elements [4], are employed to calculate normal and shear stresses of different multi-span CLT beams. The results are compared with a 2d-FE solution and the mostly used methods in practice.

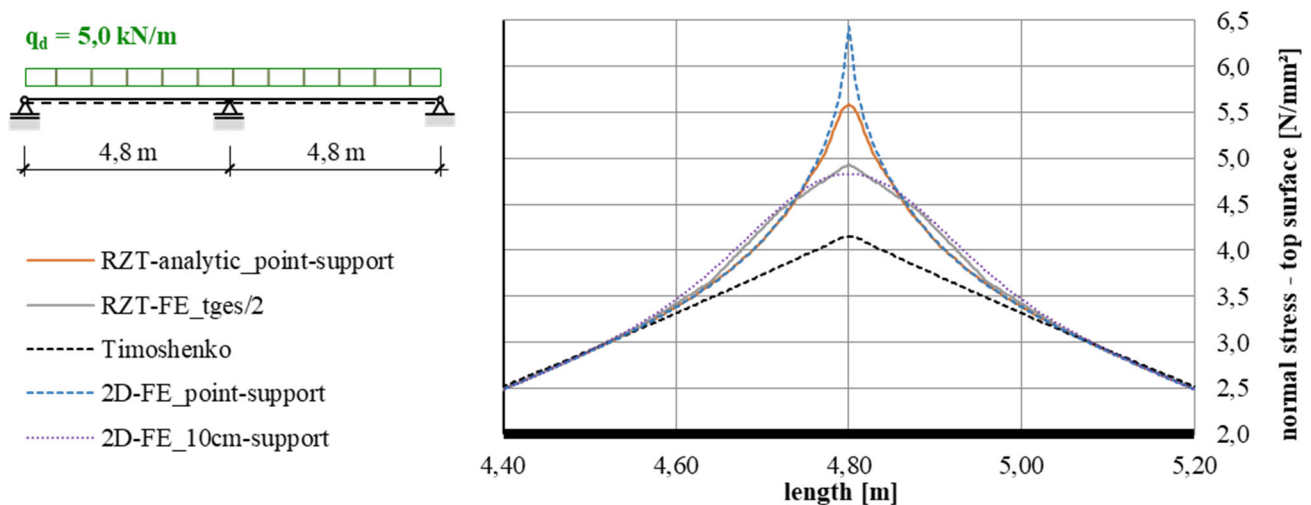


Figure 1: normal stresses on top surface of 5-layer CLT element at the middle support

Results show, that the refined zigzag theory is able to approximate the stresses within CLT elements at middle supports. An additional advantage of RZT is, that no shear correction factors are needed to calculate the deflections. Furthermore, the implementation of the employed linear FE-beam-elements based on RZT are presented and discussed.

## References

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