APPLICATION OF GEOSTATISTICAL METHODS TO DESCRIBE VARIATION IN TECHNOLOGICAL PARAMETERS OF ASH-SLAGS

Barbara Namysłowska-Wilczyńska
Wrocław University of Technology, Poland
Bożena Skorupska, Andrzej Wieniewski
Institute of Non-Ferrous Metals, Gliwice, Poland

ABSTRACT

Spatial analyses were carried out to develop a 3D geostatistical model of the spatial variation in the technological parameters of an ash-slag heap connected with the operation of the power utilities located near Świecie (Toruń). Tank A filled with ash-slags was the subject of geostatistical studies. The spatial variation of organic carbon content C_{org}, ash content A_O, moisture content W and recoverability e' (defined as the amount of carbon [kg] recovered from 1 ton of ash-slag), was analyzed. The variation of these parameters in the horizontal and vertical extents of the ash-slag heap was subjected to structural (variographic) analysis using the directional variogram function. The parameters of geostatistical models approximating the empirical variograms of content $C_{org.}$, ash content A_O, moisture content W and recoverability e' were the basis for the estimation. Averages Z* and corresponding standard estimation deviations σ_k were estimated for the particular depths of the tank using (3D) ordinary (block) kriging.

The variation of the parameters clearly differs between the particular depth levels of the waste heap. Carbon content $C_{org.}$ and recoverability e' were found to vary greatly while the variation in ash content A_O and moisture content W was respectively small and average. There is a spatial correlation between the values of the investigated parameters. The variation of content $C_{org.}$, moisture content W and recoverability e', shows a certain regularity within the waste heap tank. Maximum and minimum averages Z* (the upper limits) consistently decrease (a downward trend) with depth, in the interval of 1-10 m. The ranges of maximum averages Z* of content A_O notably increase, reaching higher values in deeper ash-slag horizons.

Thanks to the application of the geostatistical methods to describe the parameters of the waste material (treated as an anthropogenic deposit) databases containing complete valuable input information for a project aimed at utilizing the deposited material were created.

KEY WORDS

Ash-slags; Technological parameters; Variation; Spatial analyses; Geostatistical methods; Directional variogram; Ordinary kriging.

1 INTRODUCTION

The aim of the spatial analyses was to create a 3D geostatistical model of the variation in the technological parameters of an ash-slag heap (tank A) connected with the operation of the

https://doi.org/10.15626/Eco-Tech.2010.065

power utilities located in the Świecie area near Toruń in the Kujavian-Pomeranian Province. The various (estimation, simulation) applications of geostatistical methods in many fields of science and research, such as Earth sciences (geology, geophysics) and mining [5 - 6, 9 - 10] environmental protection [3 - 4, 8], climatology [8, 10], agricultural sciences, geochemistry, oceanology, epidemiology, meteorology, forestry, materials science and power engineering [5], and also to solve interdisciplinary problems have been widely reported in the literature [3 - 6, 8 - 9]. The great potential of geostatistics, enabling the modelling, estimation or forecasting of estimated averages Z^* and simulated values Z_s of the parameters characterizing various regionalized phenomena [1–2, 5, 8, 10], and the universal methodological approach of geostatistics encourage researchers to try these techniques to solve problems in new research areas. A valuable advantage of geostatistical methods is that averages Z^* and values Z_s are determined with respectively minimum estimation variance σ^2_k and also many other parameters describing the effectiveness and quality of the estimations are obtained.
The present studies of the variation in the technological parameters of the ash-slags were carried out in collaboration of Author with the Institute of Non-Ferrous Metals in Gliwice [7] as part of Project ITI (KB/140/12504/IT1-B/U/08): "Development of a technology for the utilization of waste dump ash-slags through carbon separation by the froth flotation method" pursued by the Raw Mineral Materials Processing and Waste Utilization Department in INFM [11, 12].
Only selected results of the spatial analyses of the ash-slag heap parameters are presented. Variograms of organic carbon content C_{org} and recoverability e' were modelled using the directional variogram function. Then averages Z^* of the parameters were estimated using ordinary (block) kriging. The results of the geostatistical studies are shown against the results of the estimation of the basic statistics, i.e. ash content A_O, carbon content $C_{org.}$, moisture content W and recoverability e' of the ash-slags, and the correlations between the parameters.

2 OBJECT AND RANGE OF STUDIES

The object of the statistical and geostatistical studies [1)] was tank A within the ash-slags heap [7]. The ash-slag heap tank dimensions were: 150 m x 400 m x 10 m. The specific gravity of the material in the tank was 1.948 g/cm^3. Test samples were taken from exploratory boreholes drilled within the ash-slag heap area at a depth of 1 ÷ 10 m and sample size n = 120.
Ash content A_O, moisture content W, recoverability e' were determined by the INFM Laboratory of the Raw Mineral Materials Processing and Waste Utilization Department while carbon content $C_{org.}$ was determined by the INFM Analytical Chemistry Laboratory [7]. A database (in 3D) on the four technological parameters of the ash-slags deposited in tank A was created. The database contains values of coordinates X, Y, Z (depth) specifying sampling sites and the studied regionalized variables. The database content was the basis for the spatial analyses.
The variation of the selected waste heap parameters, i.e. organic carbon content $C_{org.}$ and recoverability e'(defined as the amount of carbon [in kilograms] recovered from 1 ton of ash-slag), was studied. Parameter e' is calculated as a product of the yield and the carbon content in a given sample. The basic statistics and the distribution histograms of the parameters were estimated and the correlations between their values were analyzed.

[1)] The statistical and geostatistical analyses were carried out using selected computing programs included in the ISATIS software package – version 10.0.3, dongle for Isatis (Geovariances & Ecole Des Mines de Paris w Avon Cedex, France) (Isatis 2001).

3 INITIAL EVALUATION OF BASIC STATISTICS OF TECHNOLOGICAL PARAMETERS

The estimates of the basic statistics of the technological parameters for tank A, based on the original data, show that the behaviour of the statistics varies depending on the parameter (*see Tables: 1 ÷ 2*). Ash content A_O varies slightly (coefficient V – 8 %), moisture content W varies moderately (V – 25 %), whereas organic carbon $C_{org.}$ and recoverability e' (V- 53 ÷ 59 %) vary greatly.

4 ANALYSIS OF CORRELATIONS BETWEEN TECHNOLOGICAL PARAMETERS

The obtained linear correlation coefficients *r* indicate that the values of the investigated parameters are correlated [7]. The spatial correlations vary within tank A where the ash-slags are deposited.

Table 1. Basic statistics of technological parameters of ash-slags in waste heap

Analyzed parameter	Sample size n	Minimal value X_{min} [%]	Maximal value X_{max} [%]	Mean value X [%]	Standard deviation S [%]	Variation coefficient V [%]
ash content A_O	120	65,14	95,78	84,02	6,49	8,00
organic carbon content $C_{org.}$	120	2.05	27.43	10.02	5.31	53.00
moisture content W	120	8.84	36.43	23.64	6.01	25.00

Table 2. Basic statistics of technological parameters of ash-slags in waste heap

Analyzed parameter	Sample size n	Minimal value X_{min} [kg/$t_{mat.}$]	Maximal value X_{max} [kg/$t_{mat.}$]	Mean value X [kg/$t_{mat.}$]	Standard deviation S [kg/$t_{mat.}$]	Variation coefficient V [%]
recoverability e'	120	7.97	254.33	85.42	50.43	59.00

Strong inverse negative-sign correlations between: ash content A_O and organic carbon content $C_{org.}$ (r = - 0.75), ash content A_O and moisture content W (r = - 0.75) and ash content A_O and recoverability e' (r = - 070) were found. The strongest spatial positive-sign correlation r was between carbon content $C_{org.}$ and recoverability e' (r = 0.98).

5 STRUCTURAL ANALYSIS OF VARIATION IN TECHNOLOGICAL PARAMETERS

The structure of the spatial variation in the technological parameters was analyzed using directional variogram function γ(h) (variographic analysis). The variograms of the parameters, calculated along the borehole sampling depth, indicate a clear upward trend for variogram function γ(h) [7]. The strongest upward trend of the values of γ(h) with depth was observed in the content $C_{org.}$ variograms (*see Figure 1*) and the recoverability e' variograms (*see Figure 2*). The variograms were approximated with a spherical model ($C_{org.}$,e') (*see Tables 3 ÷ 4*) or a spherical model combined with the nugget effect C_O (W, A_O) [7]. Correlation distances, i.e. ranges of influence, varied. The longest one was found for $C_{org..}$– a = 74.24 m and much shorter for: e'– a = 13.22 m, W – a = 14.42 m and A_O – a = 13.47 m and 11.26 m. No nugget effect C_O was observed in the directional variograms of the parameters ($C_{org.}$, e'), which indicates the lack of any sharp changes in the values of γ(h).

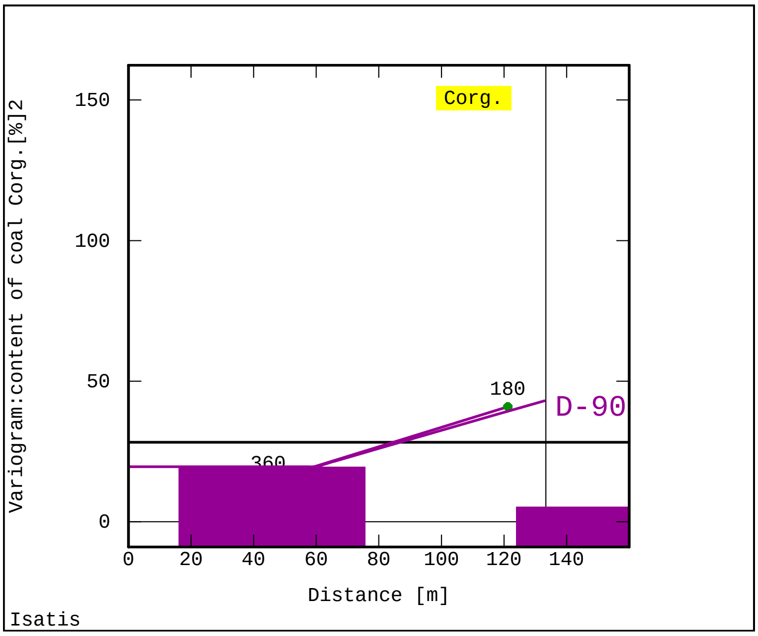

Figure 1. Directional variogram of organic carbon content $C_{org.}$ in heap ash-slags [%]^2, with the fitted theoretical model.

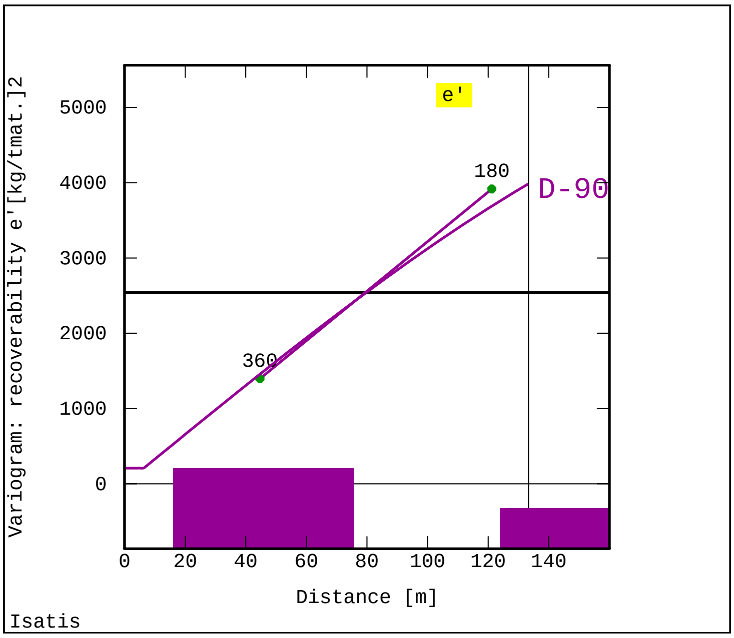

Figure 2. Directional variogram of recoverability e' of heap ash-slags [kg/t$_{mat.}$]^2, with the fitted theoretical model.

Table 3. Comparison of geostatistical model parameters of directional (along axis Z) variograms of technological parameters of ash-slags in heap (tank A)

Analyzed parameter	Nugget effect variance C_0 [%]^2	Partial sill variance C' [%]^2	Total sill variance C [%]^2	Range of influence a [m]	Principal structures of model
organic carbon content $C_{org.}$	-	-	158.74	74.24	spherical

Table 4. Comparison of geostatistical model parameters of directional (along axis Z) variograms of technological parameters of ash-slags in heap (tank A)

Analyzed parameter	Nugget effect variance C_0 [kg/t$_{mat.}$]^2	Partial sill variance C' [kg/t$_{mat.}$]^2	Total sill variance C [kg/t$_{mat.}$]^2	Range of influence a [m]	Principal structures of model
recoverability e'	-	-	5558.87	13.22	spherical

6 RESULTS OF ESTIMATION OF TECHNOLOGICAL PARAMETERS

In the next stage of the geostatistical studies the ordinary (block) kriging technique was applied. Averages Z* and standard estimation deviation σ_k were estimated in 3D for the particular ash-slags deposition depths within the heap, i.e. in an interval of 1 ÷ 10 m [7]. A 3D elementary grid with a 5 m x 5 m x 1 m elementary block was assumed for the kriging computations. The number of nodes along the particular axes was: X – 63, Y - 59 and Z – 10. The total number of 34090 nodes were taken into account in the estimation.
The kriging calculations took into account the values of the parameters of the geostatistical models representing the directional variograms of organic carbon content $C_{org.}$ and recoverability e' in the heap (*see Tables 3 ÷ 4*).
As a result of the computations new databases containing 34090 grid data on grid node coordinates X, Y and Z, averages Z* and estimation deviations σ_k, evaluated in the centres of the elementary grid blocks, were obtained [7]. The grid data can be further processed and they constitute a basis for more 3D analyses and the visualization of results and grid cross sections.

6.1. Results of estimating averages Z* of organic carbon content $C_{org.}$ in nodes of 3D elementary grid

The histogram of estimated averages Z* of organic carbon content $C_{org.}$ shows a clear tendency towards positive skewness (*see Figure 3*), with coefficient g_1 amounting to 0.75.
Variation coefficient V of averages Z* and standard estimation deviation σ_k of carbon content $C_{org.}$ in the ash-slags indicates respectively moderate (the upper limit of coefficient V) and low variation of the two geostatistical parameters (*see Table 5*) [7].

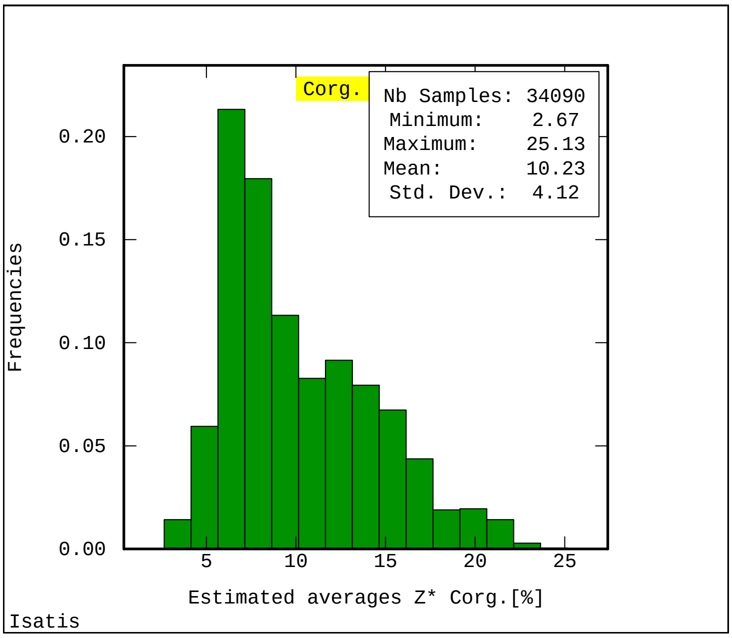

Figure 3. Histogram showing distribution of averages Z of organic carbon content $C_{org.}$ [%] in heap ash-slags.*

Table 5. Global statistics of geostatistical parameters of organic carbon content $C_{org.}$ in heap ash-slags (tank A), in nodes of 3D of elementary grid (ordinary block kriging).

Geostatistical parameter	Number of grid nodes n	Minimal estimated value X_{min} [%]	Maximal estimated value X_{max} [%]	Mean value X [%]	Standard deviation S [%]	Variation coefficient V [%]
estimated average Z^*	34090	2.90	24.33	10.24	3.91	38.00
standard deviation of estimation σ_k	34090	2.10	13.62	10.71	1.13	11.00

6.2. Results of estimating averages Z* of recoverability e' in nodes of 3D elementary grid

The histogram showing the distribution of averages Z* of ash-slag recoverability e' is characterized by distinct positive skewness (*see Figure 4*), with coefficient g_1 reaching 0.80. The coefficients of variation (V) of averages Z* indicate high variation of parameter e', similarly as previously observed for content $C_{org.}$ (*see Table 5*), and low variation of standard estimation deviation σ_k (*see Table 6*).
The variation in averages Z* of recoverability e' and (to a lesser degree) carbon content C_{org} is higher in comparison with the very low variation in averages Z* of ash content A_O and the low variation in averages Z* of moisture content W [7].

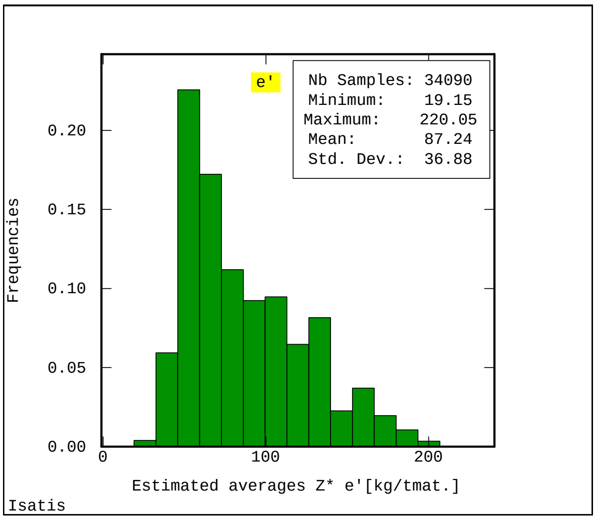

Figure 4. Histogram showing distribution of estimated averages Z of heap ash-slag recoverability e' [kg/t_{mat}].*

Table 6. Global statistics of geostatistical parameters of recoverability e' of heap ash-slags (tank A), in nodes of 3D elementary grid (ordinary block kriging).

Geostatistical parameter	Number of grid nodes n	Minimal estimated value X_{min} [kg/$t_{mat.}$]	Maximal estimated value X_{max} [kg/$t_{mat.}$]	Mean value X [kg/$t_{mat.}$]	Standard deviation S [kg/$t_{mat.}$]	Variation coefficient V [kg/$t_{mat.}$]
estimated average Z^*	34090	15.90	224.65	87.21	37.17	43.00
standard deviation of estimation σ_k	34090	20.59	145.06	113.36	13.05	12.00

7 CHARACTERIZATION AND 3D VISUALIZATION OF ESTIMATION RESULTS

The raster maps showing the distribution of estimated averages Z* and standard estimation deviations σ_k of the technological parameters of the ash-slags allow one to trace the changes in their values within the deposition area and to see if there is a spatial correlation between the values of the parameters as one moves towards the deeper horizons of the waste heap [7].

7.1. Organic carbon content $C_{org.}$ in ash-slags

The raster maps of estimated averages Z* of organic carbon content $C_{org.}$ [%] in the ash-slags are shown in *Figures 5 ÷ 6*.

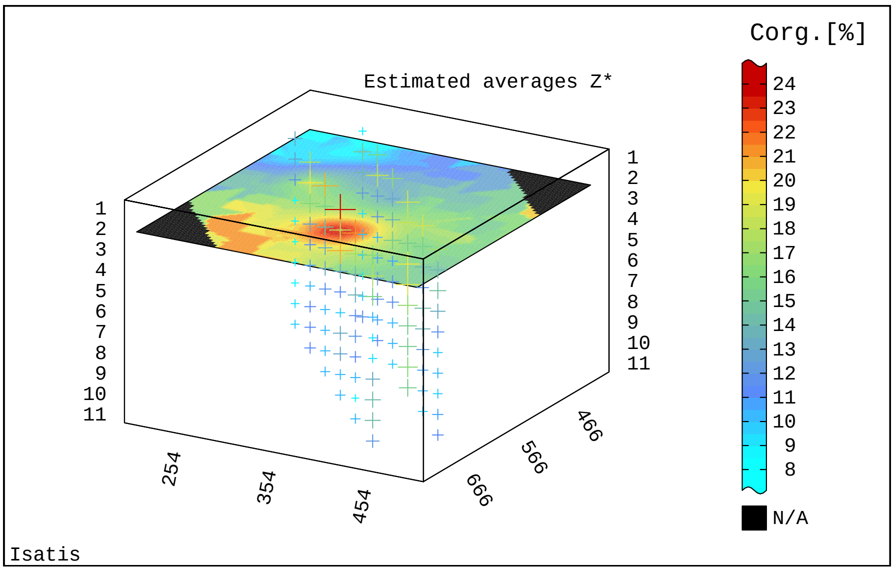

Figure 5. Raster map showing distribution of estimated averages Z of carbon content $C_{org.}$ [%] in ash-slags in heap (at depth of 2 m).*

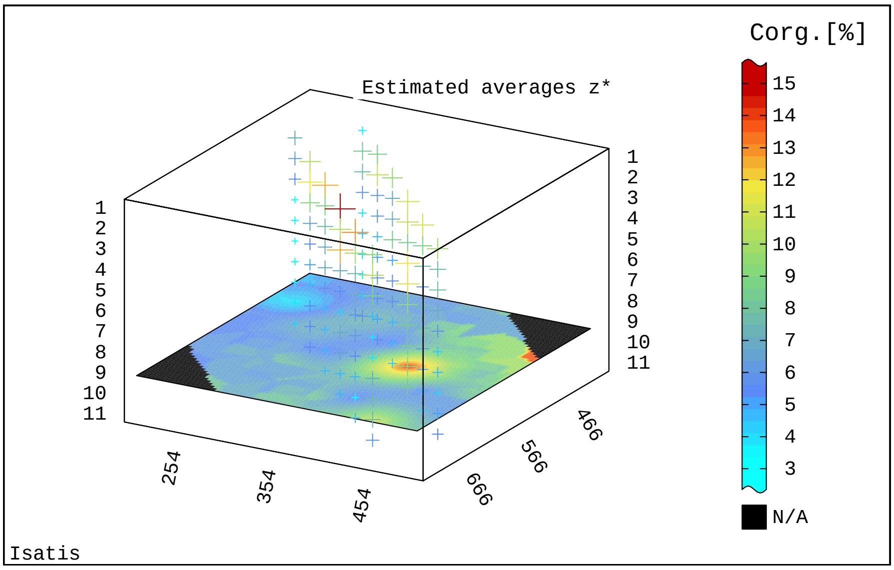

Figure 6. Raster map showing distribution of estimated averages Z of organic carbon content $C_{org.}$ [%] in ash-slags in heap (at depth of 9 m).*

In the raster maps of the distribution of $C_{org.}$ content averages Z* one can distinguish two subareas differing in the ranges of this parameter (*see Figure* 5 ÷ 6). One of the subareas is characterized by higher carbon content values in a range of 17.32 ÷ 19.11 % at a depth of 2 m (*see Figure 5)* and a narrow zone of increased $C_{org.}$ content values in a range of: 20.90 ÷ 22.68 % at the same spatial location . In the other subarea the $C_{org.}$ content is much lower, ranging from 8.37 to 11.95 % at a depth of 2 m. There is a centre of increased $C_{org.}$ content in a range of 12.54 ÷ 13.74 % at a depth of 9 m (*see Figure 6*).

The spatial location of the narrow zones and centres of increased $C_{org.}$ content exactly coincides with that of the zones of low ash content A_O while the subareas with lower $C_{org.}$ are associated with the subareas characterized by higher A_O [7].

	Maximum estimated averages Z* of organic carbon content $C_{org.}$	**Minimum estimated averages Z* of organic carbon content $C_{org.}$**
1 m	21.13 ÷ 24.33 [%]	7.22 ÷ 10.43 [%]
2 m	20.00 ÷ 22.68 [%]	8.37 ÷ 11.06 [%]
3 m	17.75 ÷ 20.26 [%]	6.89 ÷ 9.40 [%]
4 m	16.82 ÷ 19.52 [%]	5.10 ÷ 7.81 [%]
5 m	15.02 ÷ 17.82 [%]	2.90 ÷ 5.70 [%]
6 m	14.70 ÷ 17.37 [%]	3.10 ÷ 5.78 [%]
7 m	14.39 ÷ 17.04 [%]	2.92 ÷ 5.56 [%]
8 m	13.65 ÷ 15.95 [%]	3.69 ÷ 5.99 [%]
9 m	11.94 ÷ 13.74 [%]	4.16 ÷ 5.95 [%]
10 m	11.99 ÷ 13.74 [%]	4.39 ÷ 6.14 [%]

The ranges of maximum averages Z* of content $C_{org.}$ change consistently with deposition depth (*see Figures 5 ÷ 6*). A similar regularity (but with some fluctuations) is observed for the upper boundaries of minimum averages Z*.

7.2. Raster maps of estimated averages Z* of ash-slag recoverability e’

The raster maps of estimated averages Z* of ash-slags recoverability e’ [kg/$t_{mat.}$] are shown in *Figures 7 ÷8.*

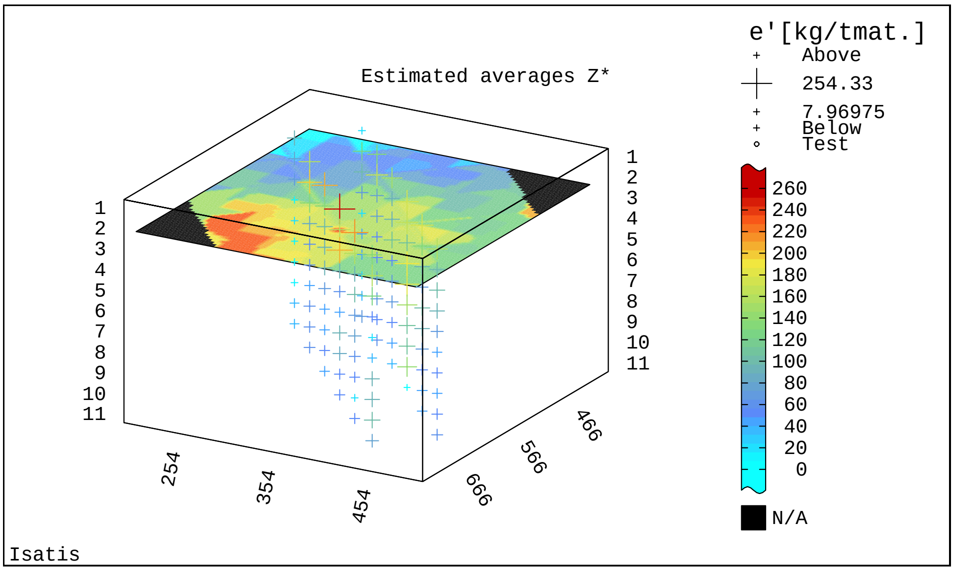

Figure 7. Raster map of estimated averages Z of ash-slag recoverability e’ [kg/$t_{mat.}$] at heap depth of 2 m.*

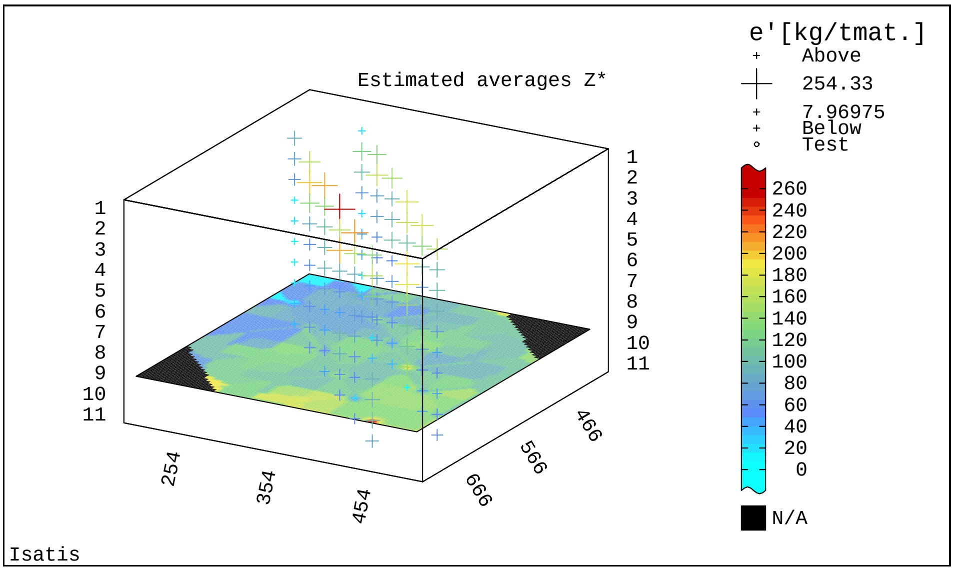

Figure 8. Raster map of estimated averages Z of ash-slag recoverability e’ [kg/$t_{mat.}$] at heap depth of 9 m.*

The raster map distributions of averages Z* of recoverability e’ (*see Figures 7 ÷ 8*) to a large degree resemble the distributions of averages Z* of carbon content $C_{org.}$ (*see Figures 6 ÷ 7*).

A small centre of increased recoverability e’ (183. 69 ÷ 210.28 kg/$t_{mat.}$) at a depth of 2 m (*see Figure 7*) can be distinguished on the raster map. Its location coincides with that of the small area of increased content $C_{org.}$ occurring at this depth (*see Figure 6*). At a depth of 9 m there is a weak zone of higher recoverability e' (Z*: 106.54 ÷ 123.02 kg/$t_{mat.}$; 106.75 ÷ 121.49 kg/$t_{mat.}$) (*see Figure 8*) , which fades out at 10 m [7].

Centres of lower ash content A_O correspond to the zones of increased carbon content $C_{org.}$ (*see Figures 5 ÷ 6*) and increased recoverability e' (*see Figures 7 ÷ 8*), and vice versa [7].

	Maximum estimated averages Z* of recoverability e'	**Minimum estimated averages Z* of recoverability e'**
1 m	192.94 ÷ 224.65 [kg/$t_{mat.}$]	55.54 ÷ 87.25 [kg/$t_{mat.}$]
2 m	183.69 ÷ 210.28 [kg/$t_{mat.}$]	68.46 ÷ 95.06 [kg/$t_{mat.}$]
3 m	158.84 ÷ 182.81 [kg/$t_{mat.}$]	54.94 ÷ 78.92 [kg/$t_{mat.}$]
4 m	150.13 ÷ 176.24 [kg/$t_{mat.}$]	36.96 ÷ 63.08 [kg/$t_{mat.}$]
5 m	136.16 ÷ 163.82 [kg/$t_{mat.}$]	16.30 ÷ 43.96 [kg/$t_{mat.}$]
6 m	125.59 ÷ 150.67 [kg/$t_{mat.}$]	16.88 ÷ 41.97 [kg/$t_{mat.}$]
7 m	123.02 ÷ 147.74 [kg/$t_{mat.}$]	15.90 ÷ 40.62 [kg/$t_{mat.}$]
8 m	121.49 ÷ 143.60 [kg/$t_{mat.}$]	25.69 ÷ 47.79 [kg/$t_{mat.}$]
9 m	77.26 ÷ 87.85 [kg/$t_{mat.}$]	31.40 ÷ 41.99 [kg/$t_{mat.}$]
10 m	73.60 ÷ 82.99 [kg/$t_{mat.}$]	32.90 ÷ 42.29 [kg/$t_{mat.}$]

The ranges of maximum averages Z* of recoverability e' show a strong tendency for this parameter to decrease with ash-slag heap depth (*see Figures 7 ÷ 8*). This observation applies to both the upper and lower limits of the averages Z*. They decrease down to a depth of 7 m, and then in the 8 ÷ 10 m interval the averages Z* increase again.

The three parameters, i.e. carbon content $C_{org.}$, moisture content W and recoverability e' show certain regularity. Maximum averages Z* and, on the whole, minimum averages Z*, and particularly their upper limits, consistently decrease. Whereas the ranges of maximum averages Z* of ash content A_O markedly increase, reaching higher values Z* in the deeper horizons.

The values of variation coefficients V of ash-slag content $C_{org.}$ (V - 38 %) and recoverability e' (V - 43 %) averages Z* indicate moderate and high variation of the parameters. The high variation of averages Z* is accompanied by relatively low values of standard estimation deviation σ_k (kriging error σ_k) and low values of V(11 – 12 %).

The highest averages Z* of content $C_{org.}$ and recoverability e' occur in the surface layers (1 - 2 m) and in the near-surface layers (3 - 4 m) of the analyzed waste heap. The deeper deposited layers (5 - 8 m) are characterized by markedly lower averages Z*. The lowest content $C_{org.}$ and recoverability e' were found in the deepest horizons (9 - 10 m). It seems that the ash-slag horizon characterized by the highest recoverability e' could be utilized by some sectors of economy (mining, power engineering).

8 CONCLUSIONS

A 3D geostatistical model representing the variation of the technological parameters of the ash-slags forming a power industry waste heap (tank A) has been developed. Thanks to this model the horizontal and vertical spatial variation of the parameters in the ash-slag heap could be accurately identified and full documentation of the identified variation, in the form of raster images of the distributions of averages Z* and standard estimation deviation σ_k, was produced.

Thanks to the application of geostatistics to the description of the variation in the technological parameters of the wastes (considered as an anthropogenic deposit) a full set of data valuable from the scientific, methodological and practical points of view was acquired. These important data can be very useful when work on the utilization of the deposited material is undertaken.

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